

CONVERGENT TWIST DEFORMATIONS

AN ANALYTIC FRAMEWORK FOR DRINFELD TWISTS

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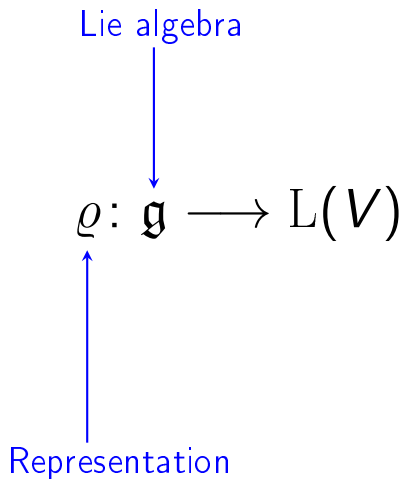
29th of April 2026

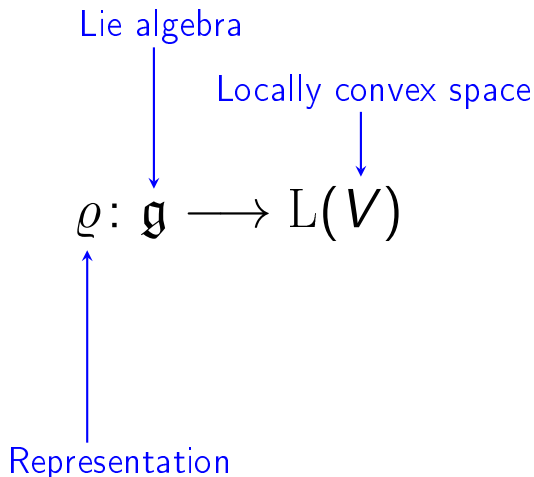
Joint work with Chiara Esposito and Stefan Waldmann

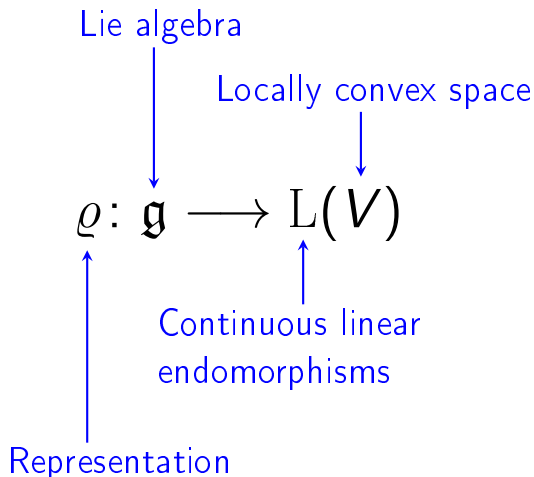
$$\varrho: \mathfrak{g} \longrightarrow L(V)$$

Lie algebra

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Act I: Algebra

Universal Enveloping algebra

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$$\Delta(x) = x \otimes 1 + 1 \otimes x$$

and

$$\epsilon(x) = 0 \quad \text{as well as} \quad S(x) = -x$$

for all $x \in \mathfrak{g} = U^1(\mathfrak{g})$.

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Get action of F , denoted by $F \triangleright \cdot$.

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$$a \star_F b := \mu(F \triangleright (a \otimes_{\mathbb{C}[[\hbar]]} b)) \quad \text{for all } a, b \in \mathfrak{A}[[\hbar]]$$

endows $\mathfrak{A}[[\hbar]]$ with the structure of a associative algebra with unit η .

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Upshot: One twist works for every representation!

- May easily be generalized to modules!

UNIVERSAL DEFORMATION FORMULA

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- Consider \mathfrak{g} -triples (V, W, X) , i.e. three representations (ρ_V, ρ_W, ρ_X) of \mathfrak{g} with a linear mapping

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- **Continuous** \mathfrak{g} -triple if μ is continuous w.r.t. projective tensor product.
- **Equivariant** \mathfrak{g} -triple if

$$\xi \triangleright \mu(v \otimes w) = \mu(\xi \triangleright v \otimes w) + \mu(v \otimes \xi \triangleright w)$$

for all $\xi \in \mathfrak{g}$, $v \in V$ and $w \in W$.

Definition (Morphisms)

A morphism between a \mathfrak{g} -triple and a $\tilde{\mathfrak{g}}$ -triple

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such that

$$T_X(\mu(v \otimes w)) = \tilde{\mu}(T_V v \otimes T_W w)$$

for all $v \in V$ and $w \in W$.

If (C, Δ, ϵ) is coassociative counital coalgebra and

$$\varrho: \mathfrak{g} \longrightarrow \text{CoDer}(C)$$

is a Lie algebra representation by co-derivations of C , then setting

$$\Delta_F := \varrho(F) \circ \Delta$$

yields a coassociative coproduct on $C[[\hbar]]$ with counit η .

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The only term containing 1 is $1 \otimes 1$, which occurs precisely once. Moreover,

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If $r = r_1 \otimes r_2$, then $(\Delta \otimes \text{id})(r) = 1 \otimes r_1 \otimes r_2 + r_1 \otimes 1 \otimes r_2$. □

Let $\mathfrak{g} = \text{span}\{H, E\}$ with $[H, E] = E$. We write

$$H_{k\uparrow} := H \cdot (H + 1) \cdots (H + k - 1)$$

with $k \in \mathbb{N} \subseteq \mathbb{C} = U^0(\mathfrak{g})$.

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A Drinfeld twist of \mathfrak{g} is given by

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with

$$F_n := \sum_{k=0}^n (-1)^k \cdot \binom{n}{k} \cdot (E^{n-k} \cdot H_{k\uparrow}) \otimes (E^k \cdot H_{(n-k)\uparrow}).$$

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Classical r -matrix $\overset{1:1}{\leftrightarrow}$ left-invariant Poisson structure on G .
 Problem: Translation fairly inexplicit.

Act II: Representation Theory

Naive Lie correspondence: $L(V)$ exponentiates into

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Extremely badly behaved group! Nevertheless,

$$\pi(\exp \xi)v := \exp(\varrho(\xi))v = \sum_{n=0}^{\infty} \frac{\varrho(\xi)^n v}{n!}$$

for all $v \in V$ and $\xi \in \mathfrak{g}$ such that the series converges.

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for all $r \geq 0$, continuous seminorms $q \in \text{cs}(V)$.

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for all $r \geq 0$, continuous seminorms $q \in \text{cs}(V)$. Write $\mathcal{E}(\varrho)$ for the space of entire vectors and endow it with locally convex topology induced by $p_{r,q}$.

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for all $r \geq 0$, continuous seminorms $q \in \text{cs}(V)$. Write $\mathcal{E}(\rho)$ for the space of entire vectors and endow it with locally convex topology induced by $p_{r,q}$.

Theorem (Lie-Taylor formula)

Get group representation $\pi: G \rightarrow \text{GL}(\mathcal{E}(\rho))$ via

$$\pi(\exp \xi) v := \exp(\rho(\xi)) v.$$

UNIVERSAL EXAMPLE

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Theorem (H., 2025)

Let G be a connected Lie group. Then

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as Fréchet algebras, where $G_{\mathbb{C}}$ denotes the universal complexification of G .

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Proposition (Esposito, H., Waldmann 2025)

Let $\pi: G \rightarrow \mathrm{GL}(V)$ be a Lie group representation of a connected Lie group with corresponding $\varrho = T_e\pi: \mathfrak{g} \rightarrow \mathcal{L}(V)$.

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Proposition (Esposito, H., Waldmann 2025)

Let $\pi: G \rightarrow \mathrm{GL}(V)$ be a Lie group representation of a connected Lie group with corresponding $\rho = T_e\pi: \mathfrak{g} \rightarrow \mathcal{L}(V)$. Then $v \in V$ is entire iff $\pi_{v,\varphi} \in \mathcal{E}(\mathcal{L})$ for all $\varphi \in V'$, where

$$\pi_{v,\varphi}: G \rightarrow \mathbb{C}, \quad \pi_{v,\varphi}(g) := \varphi(\pi(g)v).$$

Definition (Analytic vectors)

A vector $v \in V$ is called **analytic** with radius of convergence $r_0 > 0$ if

$$p_{r,q}(v) := \sum_{n=0}^{\infty} \frac{r^n}{n!} \cdot \sup_{\xi_1, \dots, \xi_n \in \mathbb{B}} q(\xi_1 \cdots \xi_n \triangleright v) < \infty$$

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A vector $v \in V$ is called **analytic** with radius of convergence $r_0 > 0$ and of **order $R \geq 0$** if

$$p_{r,q}^{(R)}(v) := \sum_{n=0}^{\infty} n!^R \cdot \frac{r^n}{n!} \cdot \sup_{\xi_1, \dots, \xi_n \in \mathbb{B}} q(\xi_1 \cdots \xi_n \triangleright v) < \infty$$

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for all $0 \leq r < r_0$ and continuous seminorms $q \in \text{cs}(V)$. Write $\mathcal{A}_{R,r_0}(\varrho)$. Define

$$\mathcal{A}_R(\varrho) := \lim_{r_0 > 0} \mathcal{A}_{R,r_0}(\varrho).$$

Typically non-strict countable inductive limit.

Definition (Analytic vectors)

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$$\mathcal{A}_R(\varrho) := \varinjlim_{r_0 > 0} \mathcal{A}_{R,r_0}(\varrho).$$

Typically non-strict countable inductive limit.

We have

$$\mathcal{E}_R(\varrho) := \varprojlim_{r_0 > 0} \mathcal{A}_{R,r_0}(\varrho).$$

Lemma

If V is Hausdorff, then so are $\mathcal{E}_R(\varrho)$, $\mathcal{A}_{R,r_0}(\varrho)$ and $\mathcal{A}_R(\varrho)$.

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Lemma

The assignment $\mathcal{A}_{R,r}: \text{Rep} \rightarrow \text{Rep}$,

$$(\varrho: \mathfrak{g} \rightarrow L(V)) \mapsto (\varrho: \mathfrak{g} \rightarrow L(\mathcal{A}_{R,r}(\varrho)))$$

acting as restriction on continuous intertwiners constitutes a covariant functor.

Lemma

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Example

Let \mathfrak{g} be abelian. Then the space $\mathcal{E}_R(\mathcal{L})$ is the space of entire functions on $\mathfrak{g}_{\mathbb{C}}$ of complex analytic order at most $1/R$.

Act III: Convergent Universal Deformations

ABSTRACT NONSENSE

Proposition

Let $\mu: V \otimes W \longrightarrow X$ be a continuous equivariant \mathfrak{g} -triple.

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$$\mu: \mathcal{E}_R(\rho_V) \otimes \mathcal{E}_R(\rho_W) \longrightarrow \mathcal{E}_R(\rho_X).$$

This yields an endofunctor.

Let $F = \sum_{n=0}^{\infty} \hbar^n / n! \cdot F_n$ be a twist and $R \geq 0$ with:

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Equicontinuity Condition

For every $r > 0$, $q_V \in \text{cs}(V)$, $q_W \in \text{cs}(W)$ and compact set $K \subseteq \mathbb{C}$

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$$\left(p_{r, q_V}^{(R)} \otimes p_{r, q_W}^{(R)} \right) \left(\frac{\hbar^n}{n!} F_n \triangleright (v \otimes w) \right) \leq C \cdot p_{tr, q'_V}^{(R)}(v) \cdot p_{tr, q'_W}^{(R)}(w)$$

for all $v \in \mathcal{E}_R(\varrho_V)$, $w \in \mathcal{E}_R(\varrho_W)$, $\hbar \in K$ and $n \in \mathbb{N}_0$.

Theorem (Esposito, H., Waldmann, 2026)

Let $\mu: V \otimes W \longrightarrow X$ be a continuous equivariant \mathfrak{g} -triple and assume the equicontinuity condition.

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Similar statement for **separate** continuity and analytic vectors.

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$$V = \bigcup_{\alpha \in J} V_\alpha$$

as vector spaces.

EVERYONE IS AN INDUCTIVE LIMIT!

Definition (Inductive)

A \mathfrak{g} -triple is called **inductive** if $V = \varinjlim V_\alpha$, $W = \varinjlim W_\beta$ and $X = \varinjlim X_\gamma$ are fully reduced inductive systems such that:

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Lemma

This implies the separate continuity of $\mu: V \otimes W \longrightarrow X$.

Let M be a smooth manifold,

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for compact sets $K \subseteq M$ are.

- Representations by differential operators leave $\mathcal{C}_K^\infty(M)$ invariant!

Definition (Analytic Vectors II)

If V is a fully reduced inductive system, then we define its space of **analytic** vectors as

$$\mathcal{A}_R(\varrho) := \varinjlim_{\alpha \in J, r_0 > 0} \mathcal{A}_{R, r_0}(\varrho_\alpha),$$

where $\varrho_\alpha := \varrho|_{V_\alpha}$.

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If the limit is strict, then this agrees with both iterative ways of taking inductive limits.

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Act IV: Convergent Universal Deformations

CONCRETE CONSIDERATIONS

Proposition (Esposito, H., Waldmann, 2026)

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Proposition (Esposito, H., Waldmann, 2026)

Let $\mathfrak{g} = \text{span}\{H, E\}$ with $[H, E] = E$ and $R = 1$. Then the Giaquinto-Zhang twist fulfils the equicontinuity condition.

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More complicated Giaquinto-Zhang twist: also $R = 1$ for all dimensions.

A CONCRETE REPRESENTATION

Let

$$\varrho(H) := -z \frac{d}{dz} \quad \text{and} \quad \varrho(E) := \frac{d}{dz}$$

acting on $V = \mathcal{H}(\mathbb{C})$, endowed with the compact-open topology.

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Proposition (Esposito, H., Waldmann, 2026)

We have $\mathcal{E}_0(\varrho) = \mathcal{H}(\mathbb{C})$ and $\mathcal{A}_1(\varrho) = \mathcal{H}_1(\mathbb{C})$ as locally convex spaces, where $\mathcal{H}_1(\mathbb{C})$ denotes the space of entire functions of complex analytic order at most one.

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Proposition (H., Roth, Waldmann, 2022)

Matrix elements of finite dimensional representations are within $\mathcal{A}_1(\mathcal{L})$.

Act V: Reaching for the Stars

WHERE DO WE GO...?

Understand the following in the analytic framework:

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- Equivalence of twists: Topology?
- Describe states. Do GNS construction.
- Relation to oscillatory integral techniques?
- Left-invariant star products.
- Quantization of symmetries to Quantum Groups.
- Locally convex Lie algebras.

LAST SLIDE.

This is the last slide. Take a look at the piece of paper or screen your notes rest on. It is probably empty. On the off-chance that it is not, the time has come to make yourself heard!